Relationship Between the Measurement and Motion Bandwidths in Magnetic Tracking

Anton PLOTKIN, Eugene PAPERNO, and Netzer MORIYA

Abstract--**The relationship between the measurement and motion bandwidths in magnetic tracking is investigated for the variety of generic sensor motion spectra. Different translation and rotation spans of the sensor motion are considered. The tracking of a three-axial magnetic sensor with a tree-axial dipole-field transmitter is chosen as the representative case. A similar behavior is expected for any other magnetic-tracking configuration based on dipole reference fields. The sensor output spectra are estimated for its generic motion spectra. Due to the nonlinearity of the representative measurement model, the sensor output spectra are wider than the sensor motion spectra. In the case where the measurement bandwidth is narrower than the sensor output spectrum, systematic tracking errors occur. In order to minimize the measurement bandwidth for the acceptable systematic tracking error, the ratio of measurement and motion bandwidths is evaluated for different translational and rotational motion spans and different tracking errors.**

***Index Terms*—magnetic tracking, measurement bandwidth, measurement model, motion spectrum, systematic errors.**

I. INTRODUCTION

MEASUREMENT bandwidth is an important characteristic of magnetic tracking systems [1–12], which are widely used to monitor moving targets in modern biomedicine, biomechanics, avionics, human-computer interface, virtual reality systems, animation, etc.

The main advantage of magnetic tracking over optical methods is the ability of monitoring hidden targets, such as laparoscopes and catheters.

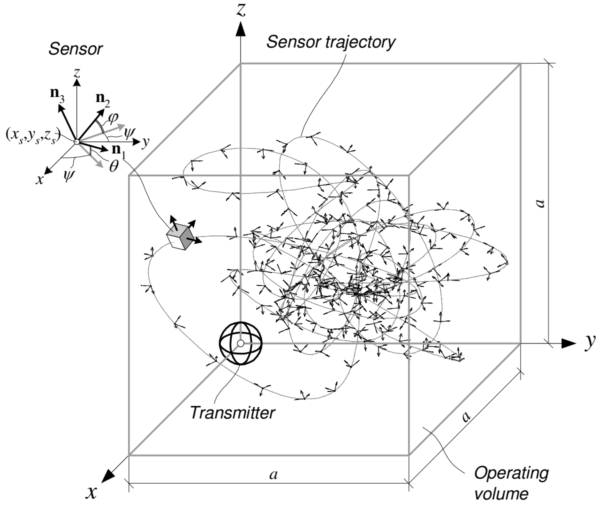

Fig. 1. A typical magnetic tracking system. The sensor motion is both translational and rotational.

Tracking the catheter position with magnetic fields allows X-ray exposure to the patient to be reduced. The fields used in magnetic tracking are of very low strength compared to magnetic resonance imaging (MRI). The magnetic tracking systems can therefore be used on patients with pacemakers, defibrillators, or other metallic implants.

A typical magnetic tracking system (see Fig. 1) consists of a transmitter and a sensor that is linked to a target to be monitored. The tracking system estimates the sensor position relative to the transmitter. This is obtained by processing the measured (sampled) sensor output with either analytical or numerical tracking algorithms.

The measurement bandwidth affects the signal-to-noise ratio (SNR) and defines the sample rate of the sensor output. In order to maximize the SNR and minimize the sample rate, the measurement bandwidth should be minimized.

It is obvious that the minimum measurement

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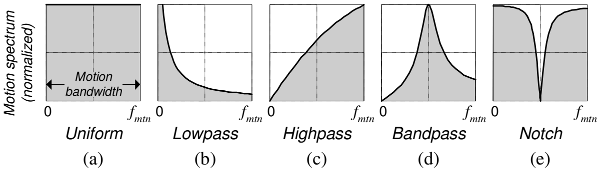


Fig. 2. Different types of average motion spectra: (a) the uniform spectrum of a finite width, $[0, f_m]$, was modified with the (c) highpass and (b) lowpass first-order Butterworth filters and the (e) notch and (d) bandpass second-order Butterworth filters.

bandwidth depends on the target motion bandwidth and the nonlinearity of the measurement model (the conversion of the sensor motion into its output).

However, it is a nontrivial task to find the *exact* relationship between the measurement and motion bandwidths. Existing literature suggests no treatment to this problem.

The aim of the present work is to bridge the above gap and provide both the designers and users of magnetic tracking systems with a comprehensive description of the minimum measurement bandwidth for a given motion spectrum and an acceptable tracking error.

In order to reach this goal, we define in Section II a variety of generic motion spectra and estimate for each of them the corresponding sensor output spectrum. We then evaluate in Section III the ratio of measurement and motion bandwidths, $B_{\mathrm{msr}}/B_{\mathrm{mtn}}$, for different translational and rotational motion spans and different tracking errors. We evaluate the above ratio separately for the location and orientation tracking errors.

II. SENSOR OUTPUT SPECTRUM

We estimate the sensor output spectra for the magnetic-tracking configuration [3]. We consider the tracking of a three-axial sensor with a tree-axial dipole-field transmitter (see Fig. 1). We expect a similar behavior for any other magnetic-tracking configuration based on dipole reference fields [3]–[12].

To estimate the sensor output spectra we use a procedure [13] for the system identification in frequency domain. This procedure is based on the averaging of a large number of output spectra realizations. In each realization, the measurement model is excited with a set of random-phase sinewaves representing a motion spectrum.

In this work, we divide all the variety of motion spectra into the five representative types shown in Fig. 2. All these types are modifications of the uniform spectrum of a finite width, $[0, f_{mtn}]$.

We are going to investigate the sensor output spectra for different motion spans. To define motion spans, we assume first that the mean sensor location is at the center of the cubic operating volume of size a (see Fig. 1), and the mean sensor orientation is described by zero rotation angles.

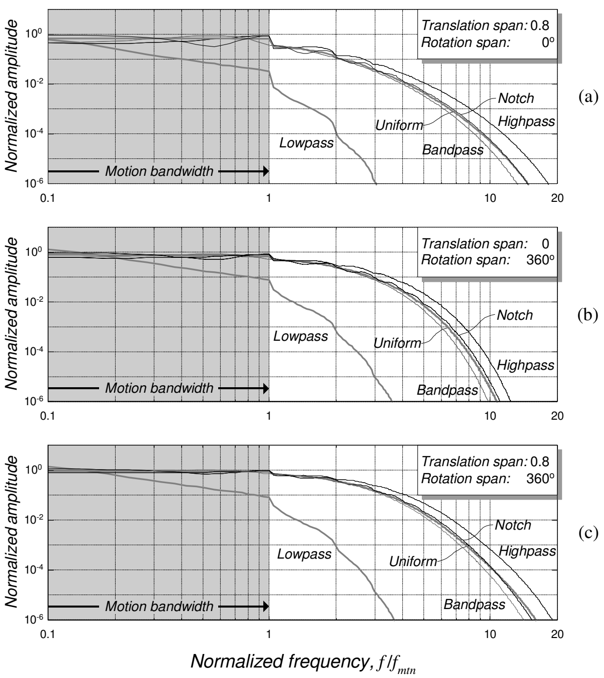
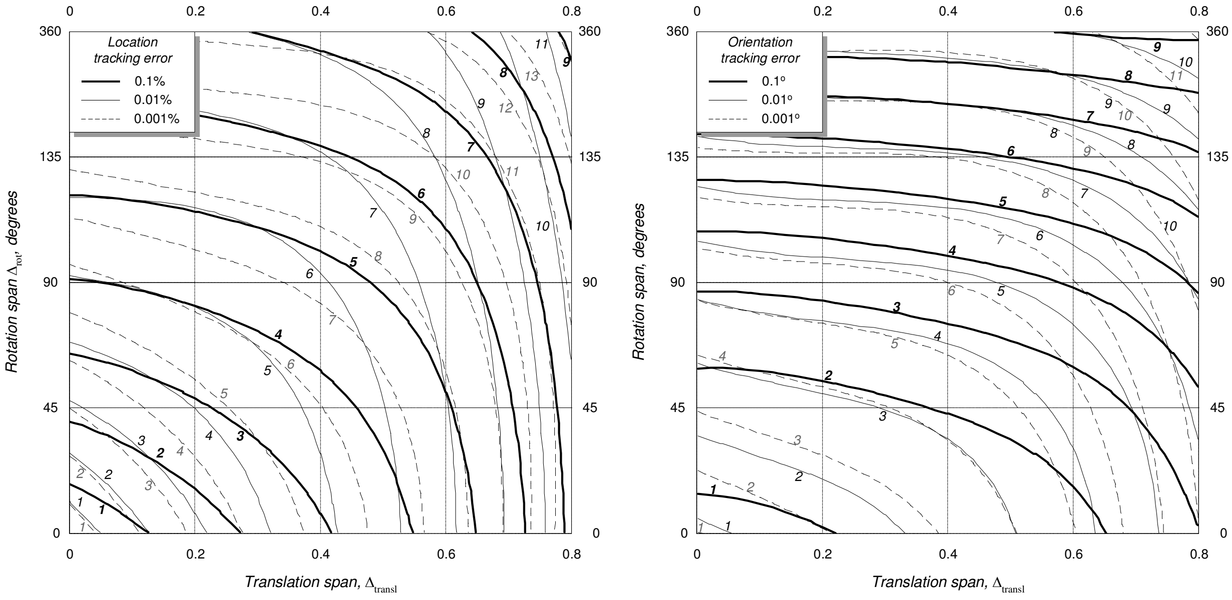


Fig. 3. The sensor output spectra for different types of motion spectra (see Fig. 2) and maximum translation and rotation spans. (a) Translational motion only. (b) Rotational motion only. (c) The motion is both translational and rotational.

Considering then a practical case, where the sensor translation and rotation are limited, we define motion spans in the following manner. We set equal spans, Δ_{transl}, for the sensor location coordinates $x_s(t)$, $y_s(t)$, $z_s(t)$. This limits sensor translation within a cubic volume around the center of the total operating volume. We also set equal spans, Δ_{rot}, for the sensor orientation angles $\psi(t)$, $\theta(t)$, $\varphi(t)$. This limits sensor rotation within equal solid angles.

To bring sensor motion in accordance with translation and rotation spans, we set the standard deviation of each sensor coordinate equal to one sixths of the corresponding span value. We disregard the realizations where sensor coordinates exceed the chosen span. Since the sensor motion in our case is a Gaussian random process, the number of disregarded realizations does not exceed 0.26%.

The output spectrum for each type of sensor motion of Fig. 2 and given translation and rotation spans has been obtained as the root-mean-square (rms) of the 10^4 output spectra realizations. We have considered translation spans from 0 to 0.8 of the operating volume



(a)

(b)

Fig. 4. Ratio of measurement and motion bandwidths for uniform spectrum as a function (contour plot) of the sensor translational and rotational motion spans. (a) Location tracking error is a fixed parameter: 0.001, 0.01, and 0.1% of $a/2$. (b) Orientation tracking error is a fixed parameter: 0.001, 0.01, and 0.1°.

size a and rotation spans from 0 to 360°.

The sensor output spectra that correspond to maximum motion spans are represented in Fig. 3. The maximum motion spans illustrate the greatest effect of sensor translation and rotation on its output spectra.

Fig. 3 shows a significant widening of the sensor output spectra due to the nonlinearity of the measurement model [3]. This effect is moderate only for the lowpass-shaped-spectrum motion.

III. RATIO OF MEASUREMENT AND MOTION BANDWIDTHS

A. Method

To find the minimum measurement bandwidth, we evaluate in this section the ratio of the measurement and motion bandwidths, $B_{\mathrm{msr}}/B_{\mathrm{mtn}}$, as a function of an acceptable systematic tracking error.

To reach this goal, we first solve the *reciprocal* problem: find the dependence of the systematic tracking error on the $B_{\mathrm{msr}}/B_{\mathrm{mtn}}$ ratio. We do this for different translation and rotation motion spans. For all the 10^4 motion realization, we find corresponding sensor output, filter it with an ideal lowpass filter of the bandwidth B_{msr}, load it into the tracking algorithm [3], and find the tracking error as the difference between the true and calculated sensor positions.

We assume that for most practical applications it is important to know the maximum tracking error. We define the maximum tracking error as that for which the probability of tracking errors to be below this maximum equals 99.9%.

Finally, we plot the $B_{\mathrm{msr}}/B_{\mathrm{mtn}}$ ratio as a function (contour plot) of the sensor translation and rotation spans, where the tracking error is a fixed parameter.

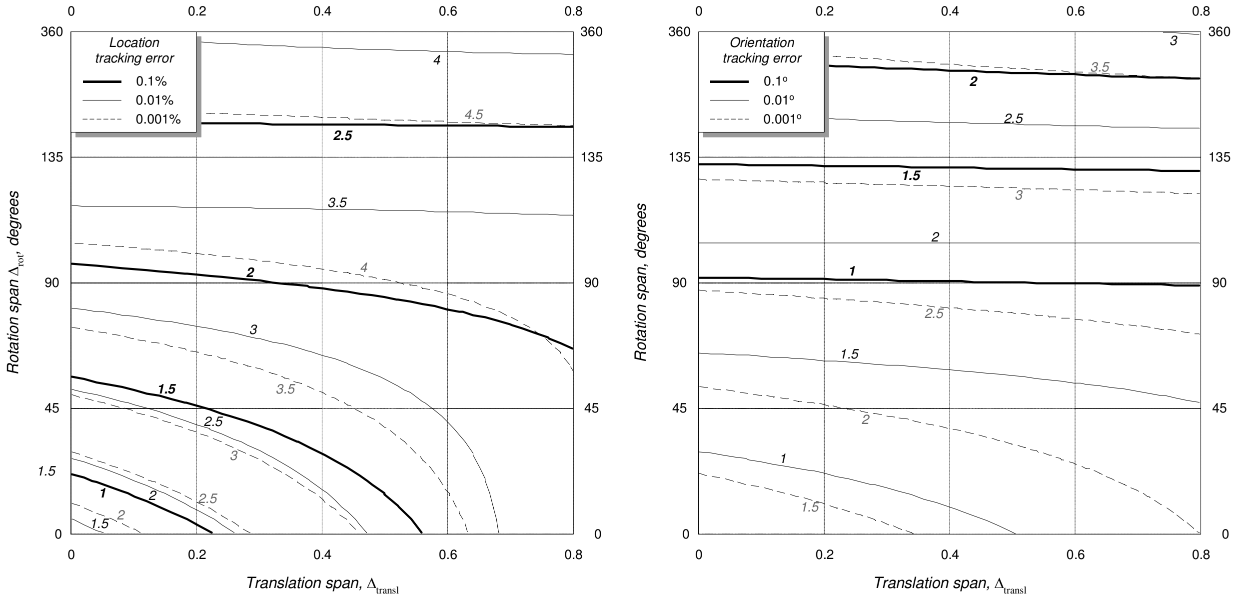
B. The B_{msr}/B_{mtn} Ratio for Different Motion Spectra

The $B_{\mathrm{msr}}/B_{\mathrm{mtn}}$ ratio for the uniform-spectrum motion is shown in Fig. 4 for 0 to 0.8 translation spans and 0 to 360° orientation spans. The location tracking error is a fixed parameter in Fig. 4(a), and the orientation tracking error is a fixed parameter in Fig. 4(b).

The small difference between the sensor output spectra in Fig. 3 allows us to describe the $B_{\mathrm{msr}}/B_{\mathrm{mtn}}$ ratios for the highpass-, bandpass-, and notch-shaped-spectrum motion types with the charts of Fig. 4 that has been obtained for the uniform-spectrum motion.

To do this, we simply find correction coefficients that are 1.2, 1.0, and 0.95 for highpass-, notch-, and bandpass-shaped-spectrum motion types, correspondingly.

We describe the $B_{\mathrm{msr}}/B_{\mathrm{mtn}}$ ratio for the lowpass-shaped-spectrum motion with the charts of Fig. 5, since the sensor output spectra for this motion type significantly differs from that for the uniform-spectrum motion.



(a) (b)

Fig. 5. Ratio of measurement and motion bandwidths for lowpass filtered uniform spectrum as a function (contour plot) of the sensor translational and rotational motion spans. (a) Location tracking error is a fixed parameter: 0.001, 0.01, and 0.1% of *a*/2. (b) Orientation tracking error is a fixed parameter: 0.001, 0.01, and 0.1°.

IV. CONCLUSIONS

The relationship between the measurement and motion bandwidths in magnetic tracking has been investigated for different sensor motion spectra types and different translation and rotation spans. The results are presented in the form of special charts, where the translation and rotation spans are variables and the acceptable tracking error is the parameter.

The measurement model [3] was chosen as the representative one. We expect similar results for any other measurement model based on dipole fields. For example, our simulations (not described in this work) for the measurement model [10] give very close results to those in Fig. 3.

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